安徽理工大學

ANHUI UNIVERSITY OF SCIENCE & TECHNOLOGY

声学晶体中的自旋拓扑相

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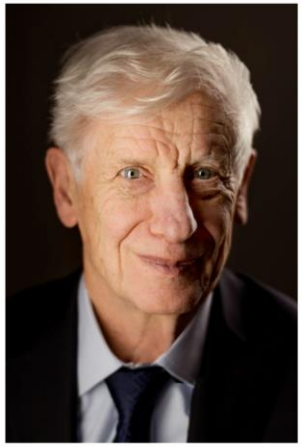
2023/11/25

Outline

- **Backgrounds of topology** → What?
- **Motivation for spinful topological phases** → Why?
- **Analytical model for spinful topological phases** → How?
- **Sample design and experimental demonstration**
- **Conclusions**

Background of topological insulators

2016 Nobel Prize



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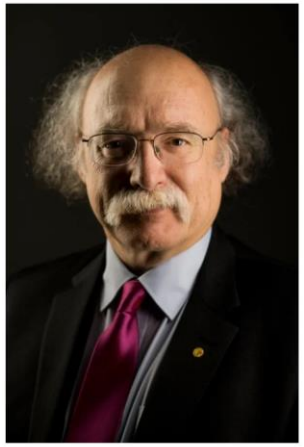
David J. Thouless

Prize share: 1/2



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F. Duncan M. Haldane

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J. Michael Kosterlitz

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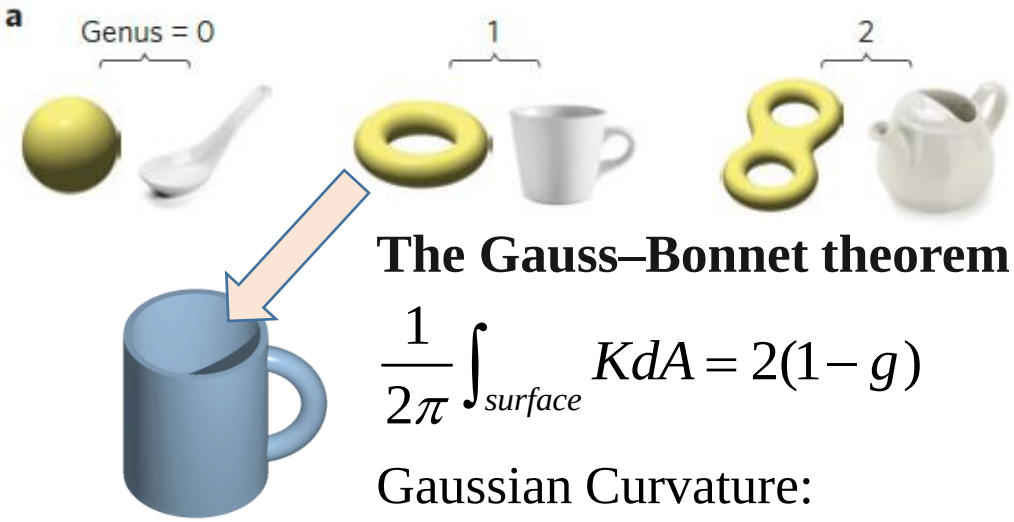
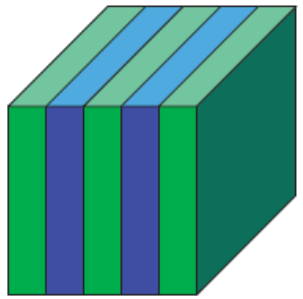


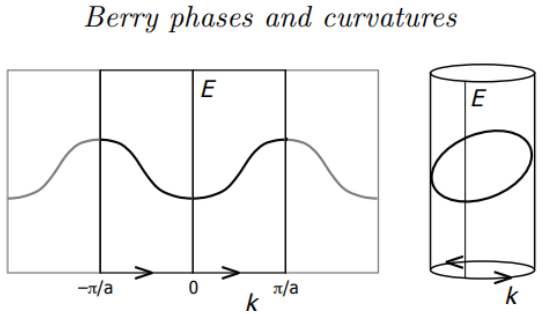
The Gauss–Bonnet theorem:

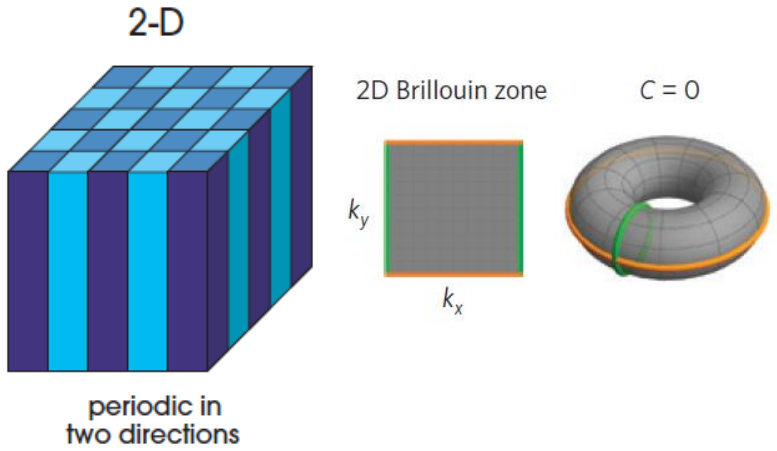
$$\frac{1}{2\pi}\int_{surface} KdA = 2(1-g)$$

Gaussian Curvature:

$K=1/(R_1R_2)$







Berry phase:

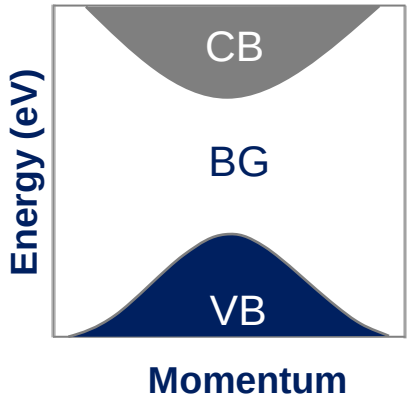
$$\gamma_C = \oint_C \mathbf{A}\cdot d\mathbf{k}$$

Berry connection

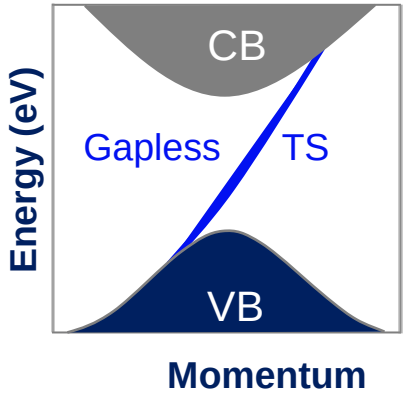
$$\mathbf{A} = -i\langle u(\mathbf{k})|\nabla_{\mathbf{k}}|u(\mathbf{k})\rangle$$

Background of topological insulators

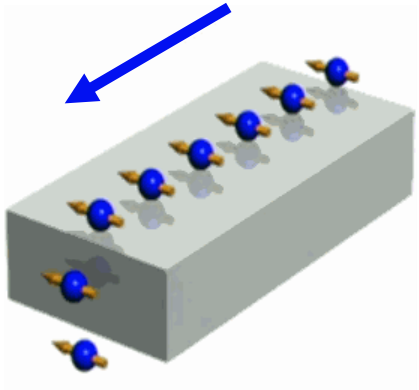
Trivial insulator



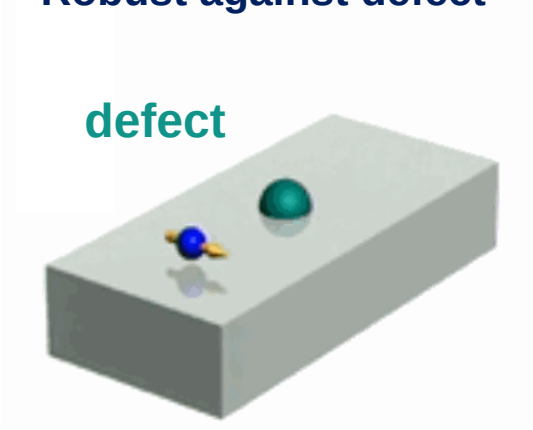
Topological insulator



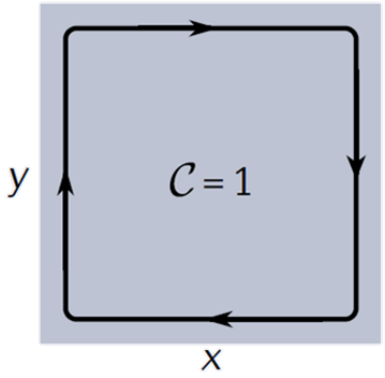
Unidirectional propagation



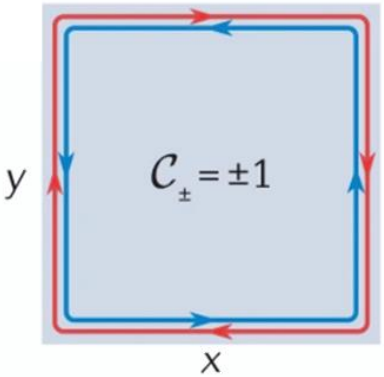
Robust against defect



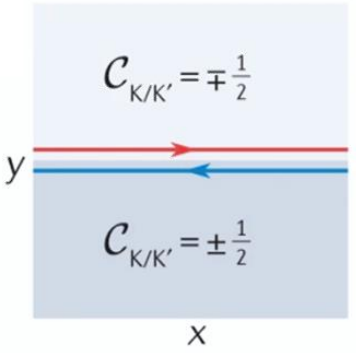
Topological Chern insulator



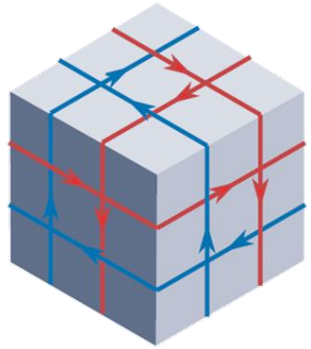
Topological spin Hall insulator



Topological valley Hall insulator

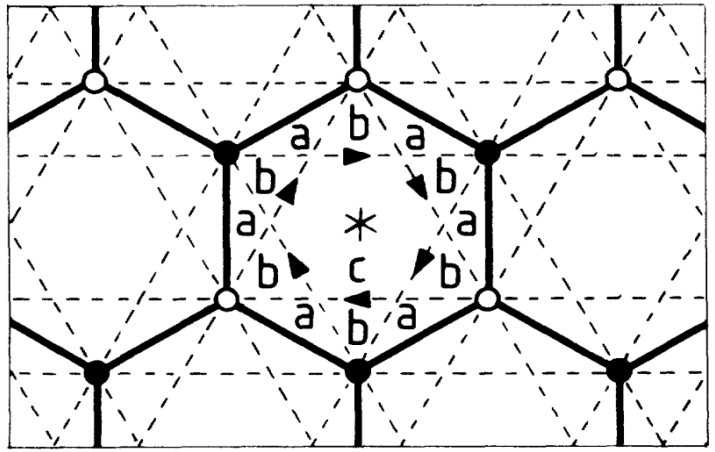


3D topological insulator



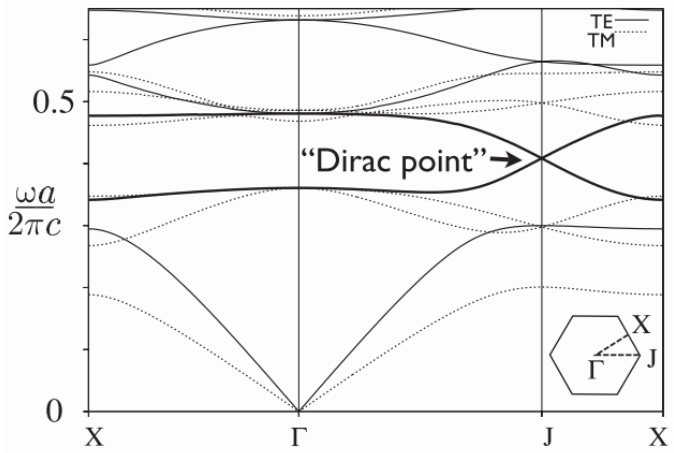
Nat. Rev. Mater. **7**, 974 (2022).

Background of topological insulators



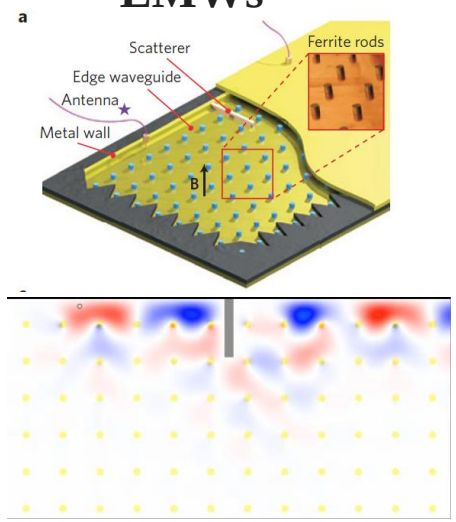
Breaks time-reversal Quantum Hall effect

Phys. Rev. Lett. **61**, 2015 (1988)



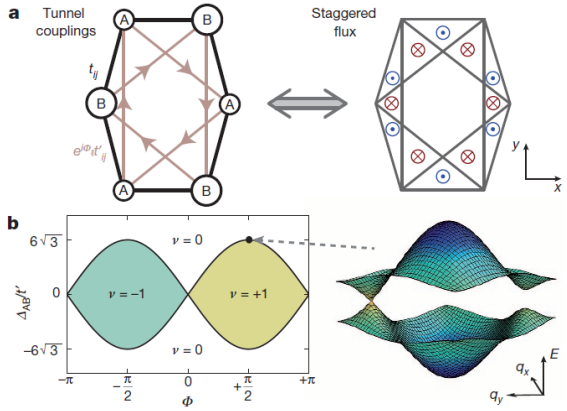
Phys. Rev. Lett. **100**, 013904 (2008)

EMWs

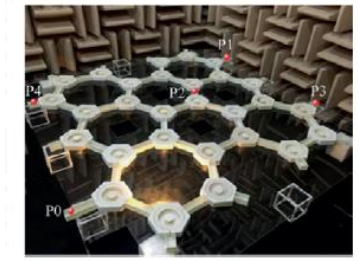


Nature **461,** 772–775 (2009)

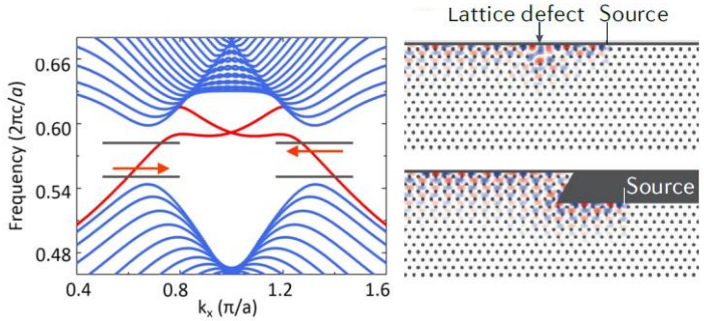
Ultracold fermions



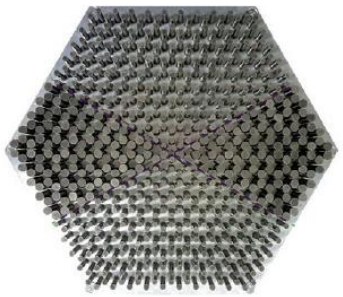
Nature, **515**, 237 (2014)

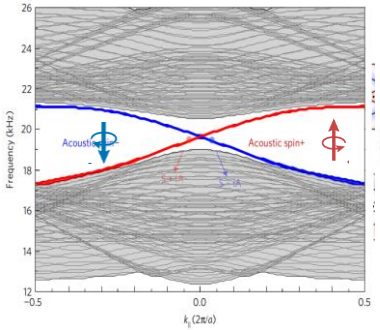


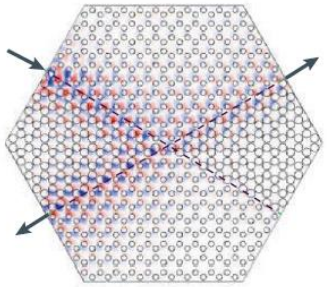
Science **343**, 516 (2014)



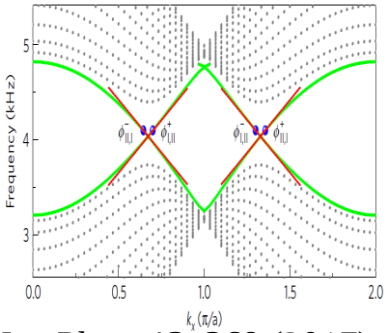
Phys. Rev. Lett. **114**, 114301 (2015)

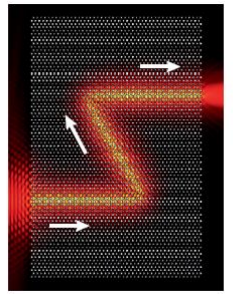






Nat. Phys. **12**, 1124 (2016)



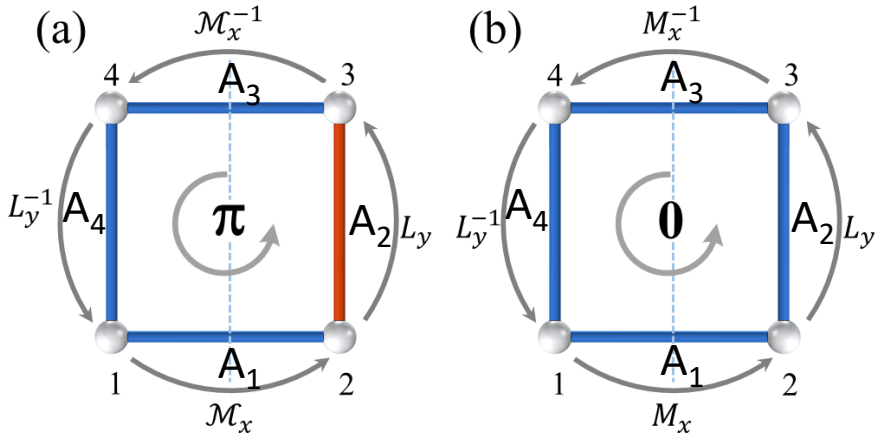


Nat. Phys. **13**, 369 (2017).

Backgrounds of projective symmetry

Gauge flux

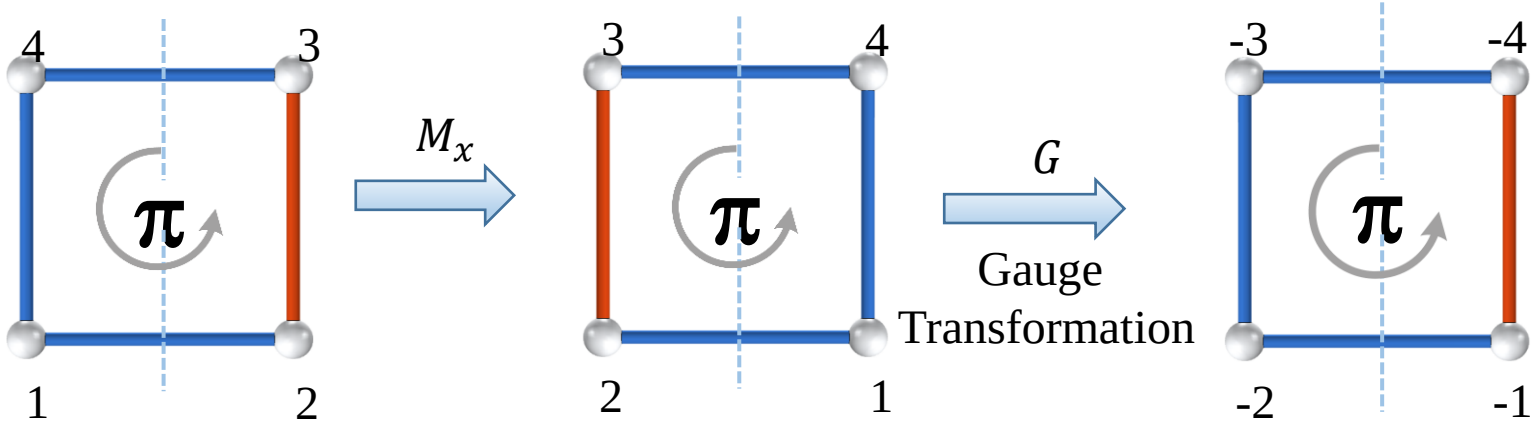
$\mathbb{Z}_2$ Gauge field → Coupling coefficients $\{-,+\}$



$e^{i\Phi} = \prod_{m \in plaquette} A_m \ (m = 1,2,3,4)$

$L_y^{-1}\mathcal{M}_x^{-1}L_y\mathcal{M}_x = e^{i\Phi}$

Site: Positive coupling: Negative coupling:



Gauge transformation:

$GH(t_x, t_y)G^\dagger = H(t_x, -t_y)$

Projective mirror symmetry:

$\mathcal{M}_x = GM_x$

$M_x^2 = \mathrm{I}; \mathcal{M}_x^2 = -\mathrm{I}$

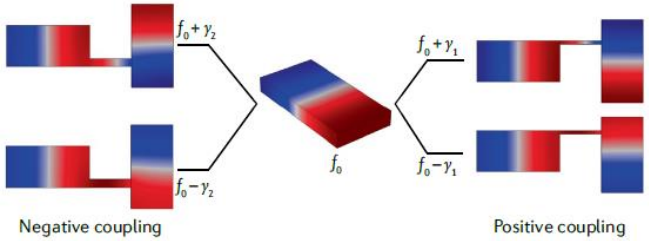
Acoustics:

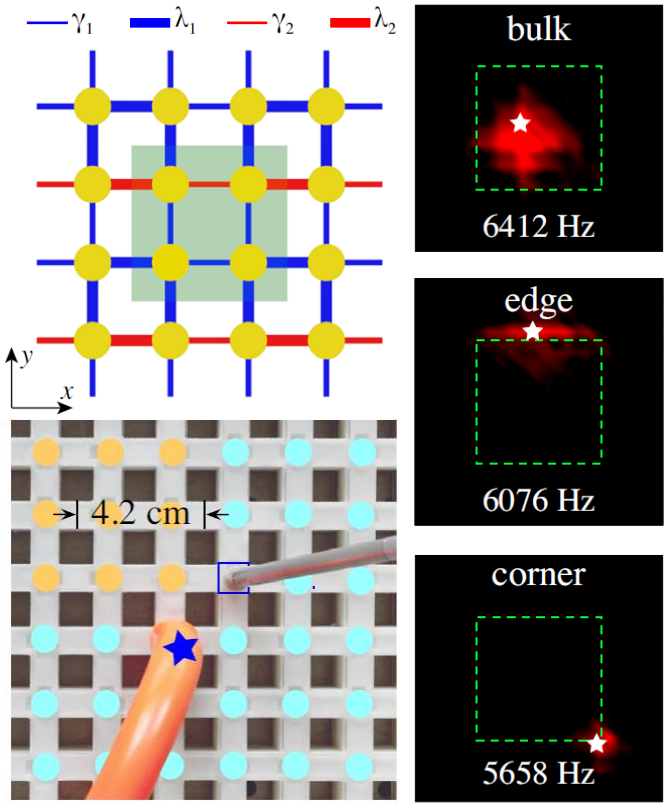
$\hat{T} = \hat{\mathcal{K}}, T^2 = 1$

Electrons:

$\hat{T} = -i\sigma_y\hat{\mathcal{K}}, \ T^2 = -1$

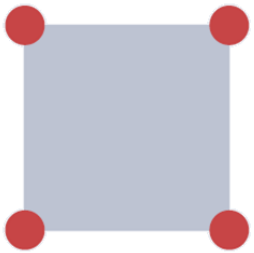
Background of topological acoustics

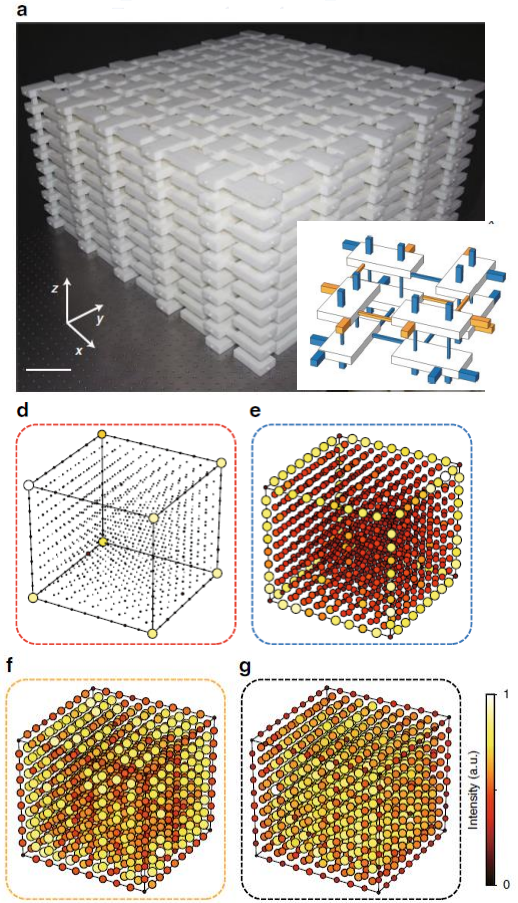




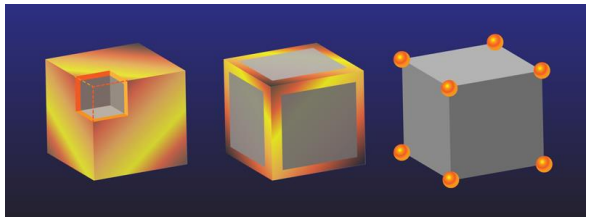
Experimental Setup Experiment

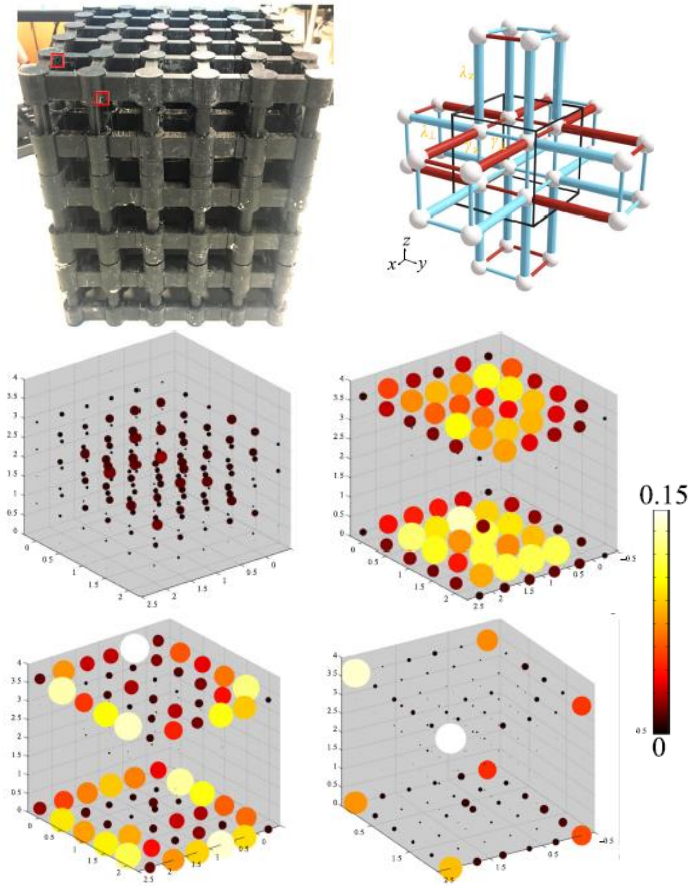
Phys. Rev. Lett. **124**, 206601 (2020)





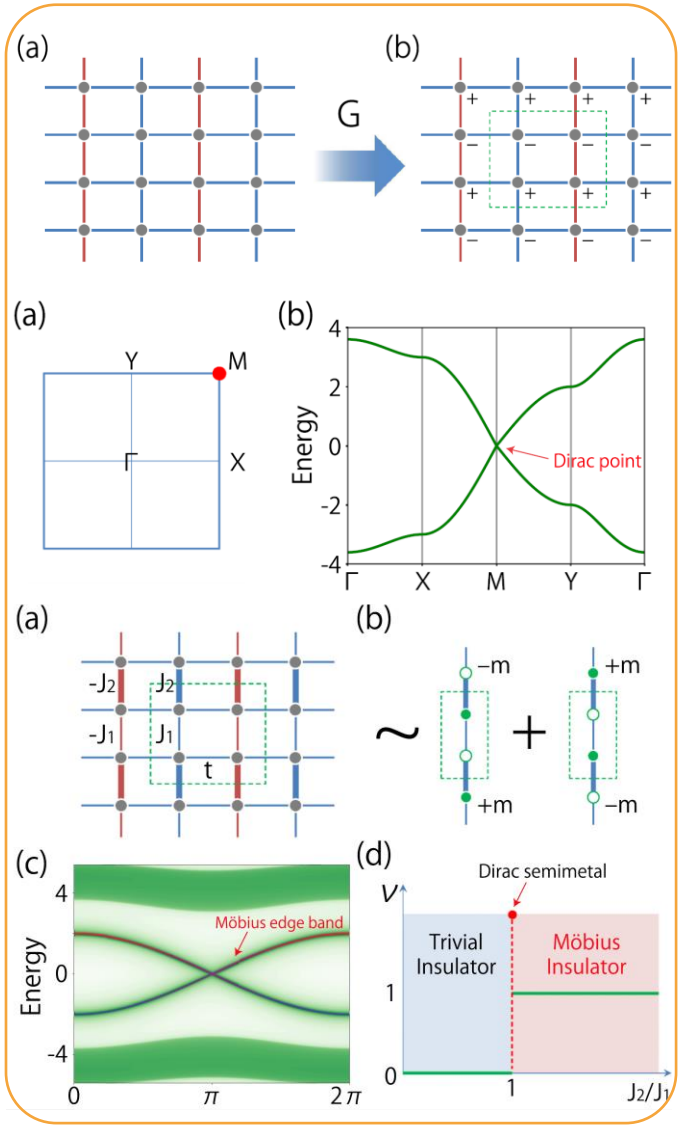
Nat. Common. **11**, 2442 (2020)



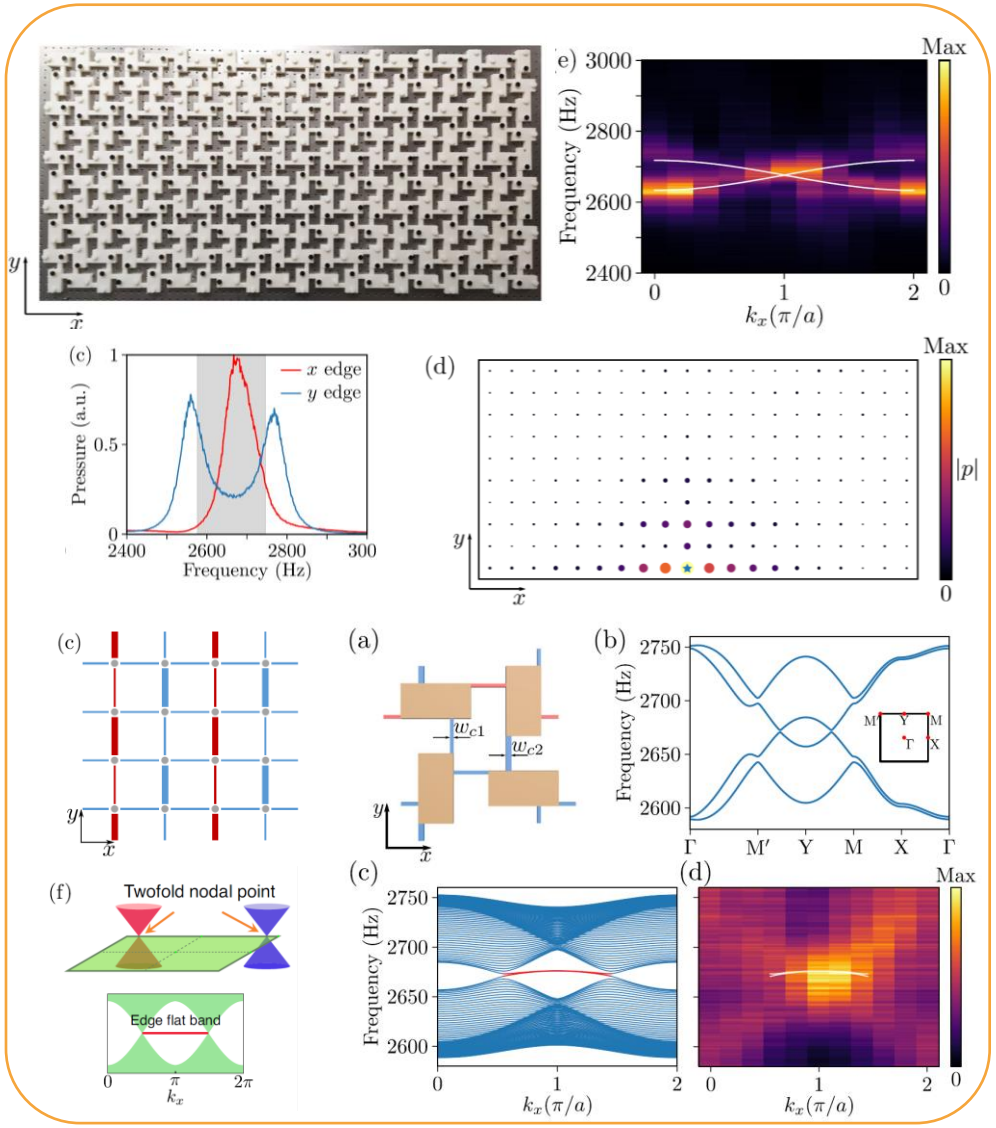


Nat. Common. **11**, 2108 (2020)

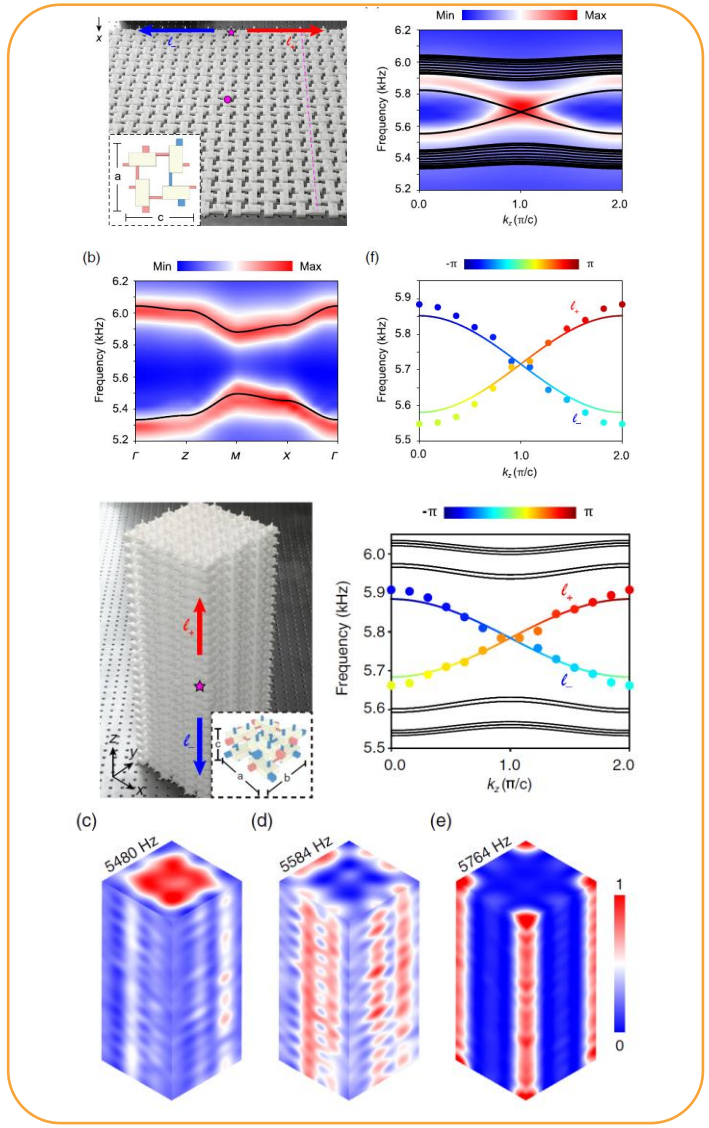
Backgrounds of projective symmetry



Phys. Rev. B, **102**, 161117(R) (2020)

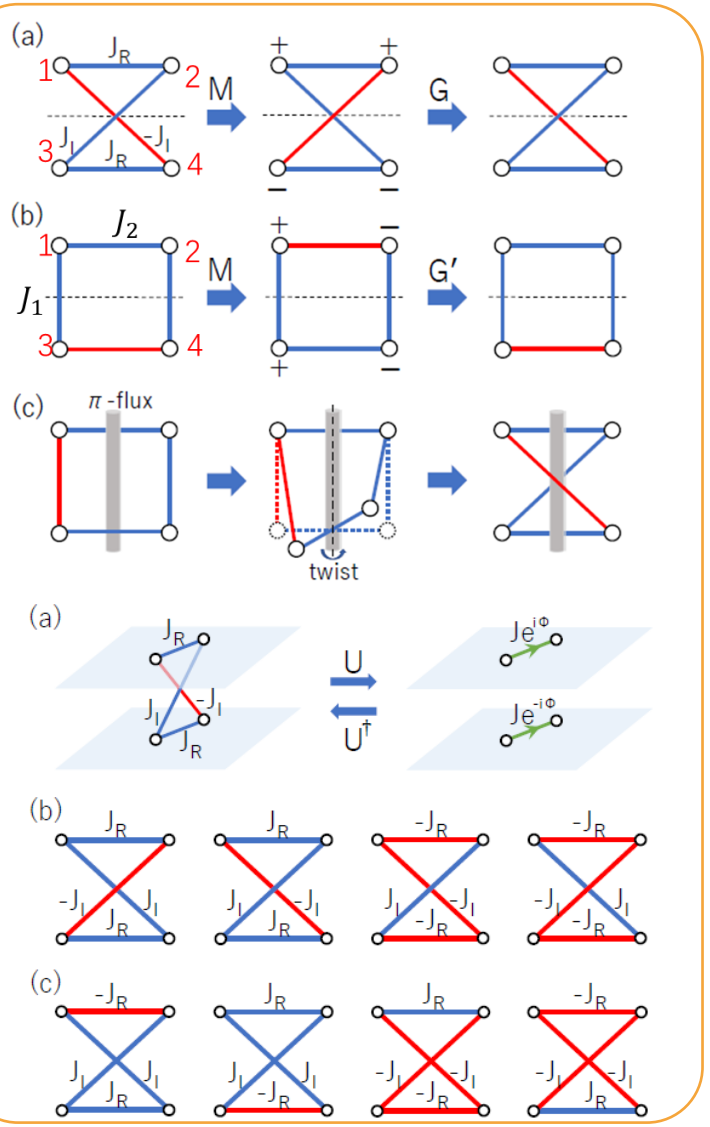


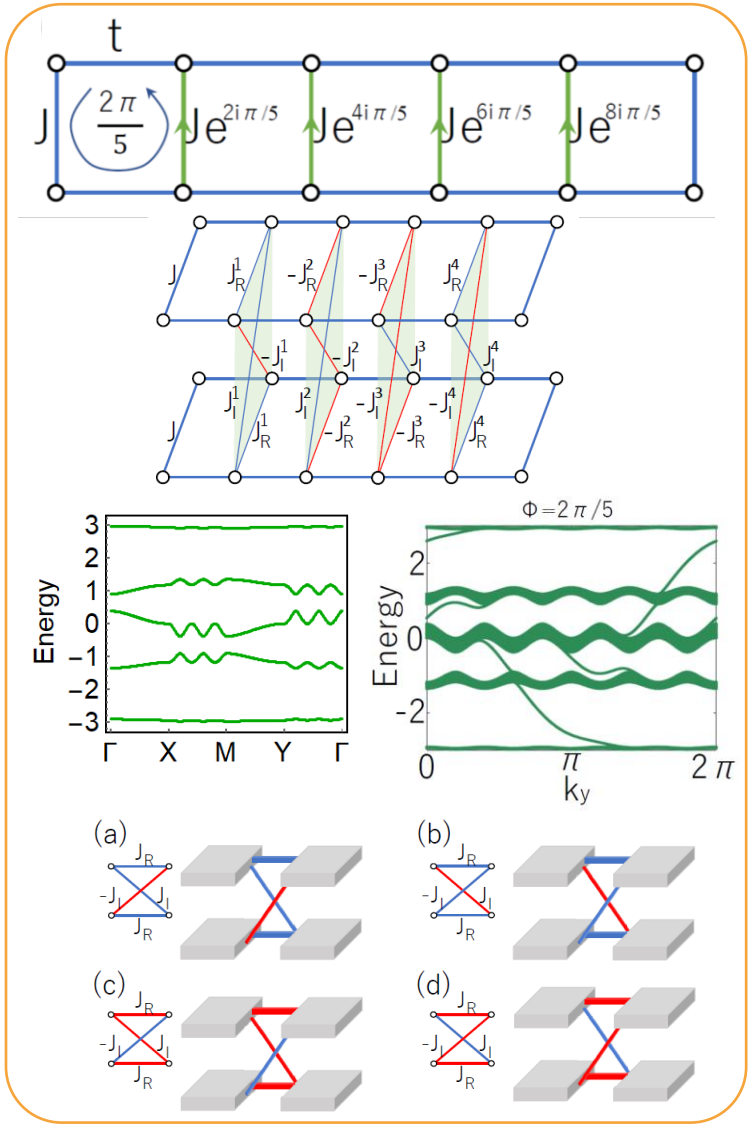
Phys. Rev. Lett. **128**, 116802 (2022)



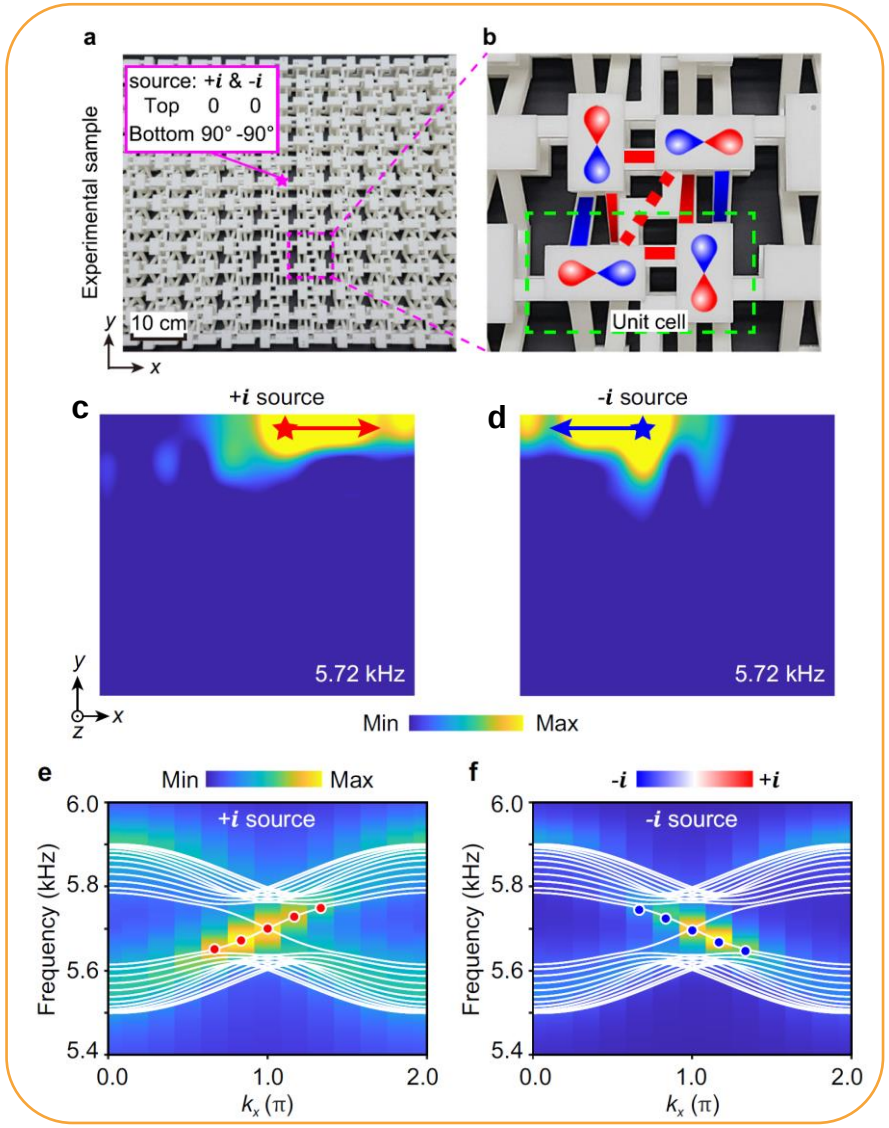
Phys. Rev. Lett. **128**, 116803 (2022)

Backgrounds of projective symmetry



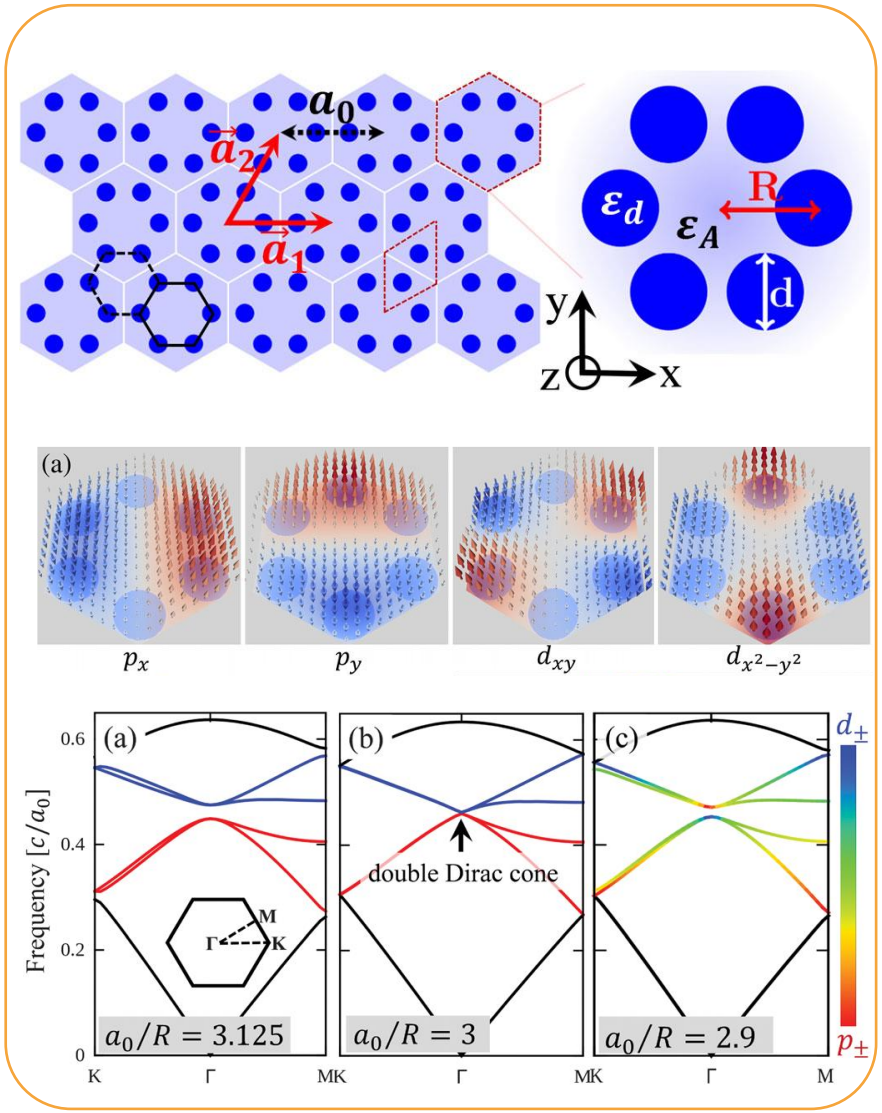


Phys. Rev. B **108** 205126 (2023)

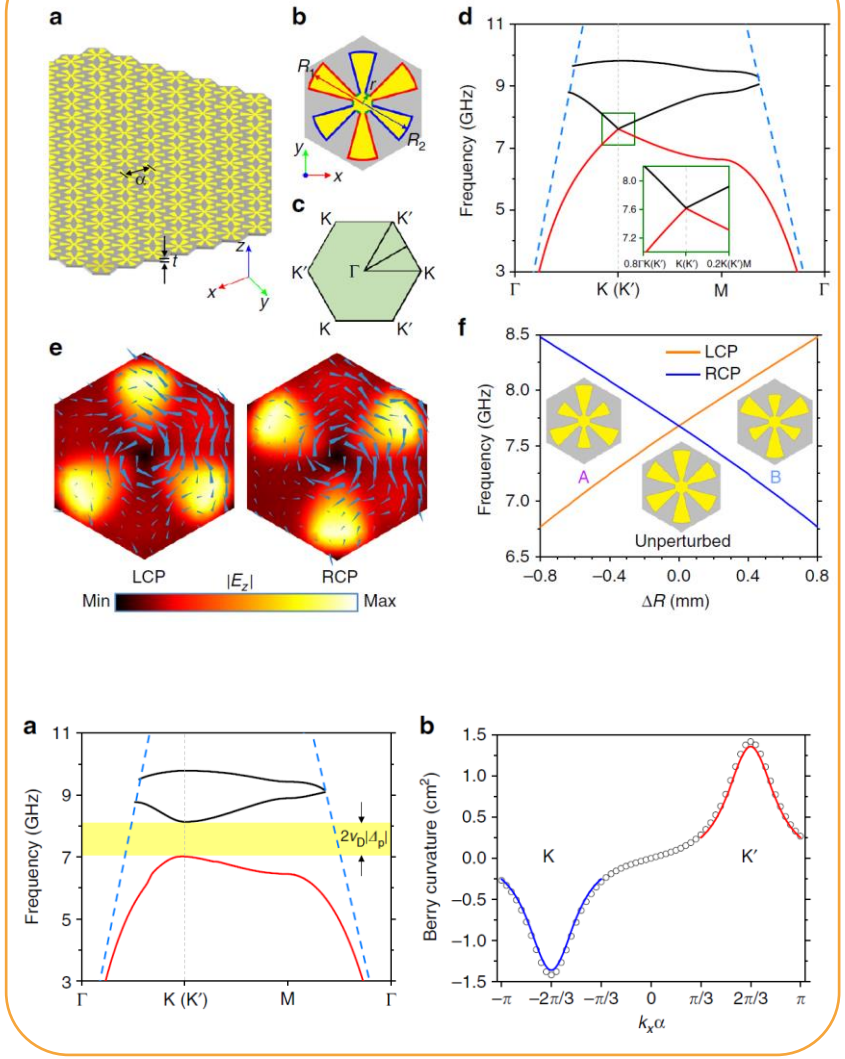


Commun. Phys. **6**, 268 (2023)

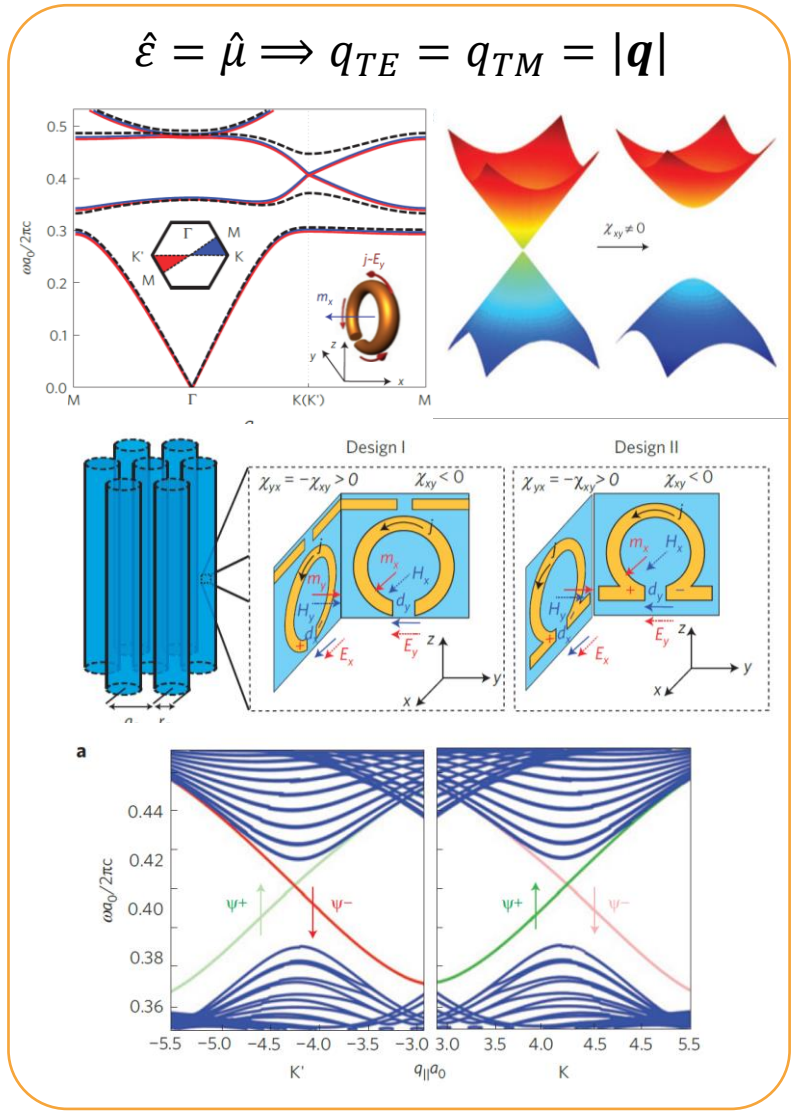
Motivation for spinful topological phases



Phys. Rev. Lett. **114**, 223901 (2015)



Nat. Commun. **8**, 1304 (2017)



Nat. Mat. **13**, 233 (2013)

Analytical model for spinful topological phases

Topological classification under spacetime inversion symmetry (PT) and sublattice symmetry (S).

	$(PT)^2$	S	$d=1$	$d=2$	$d=3$
AI	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0
BDI	+	$[S,PT]=0$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$
CI	+	$\{S,PT\}=0$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
AII	−	0	0	0	0
CII	−	$[S,PT]=0$	0	0	$\mathbb{Z}$
DIII	−	$\{S,PT\}=0$	$2\mathbb{Z}$	0	0

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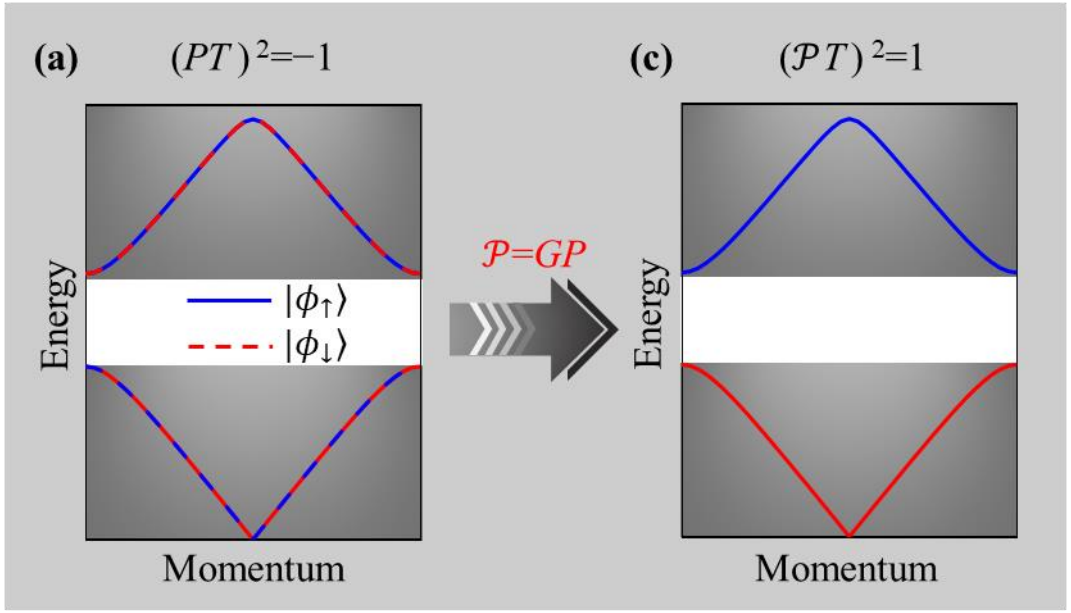
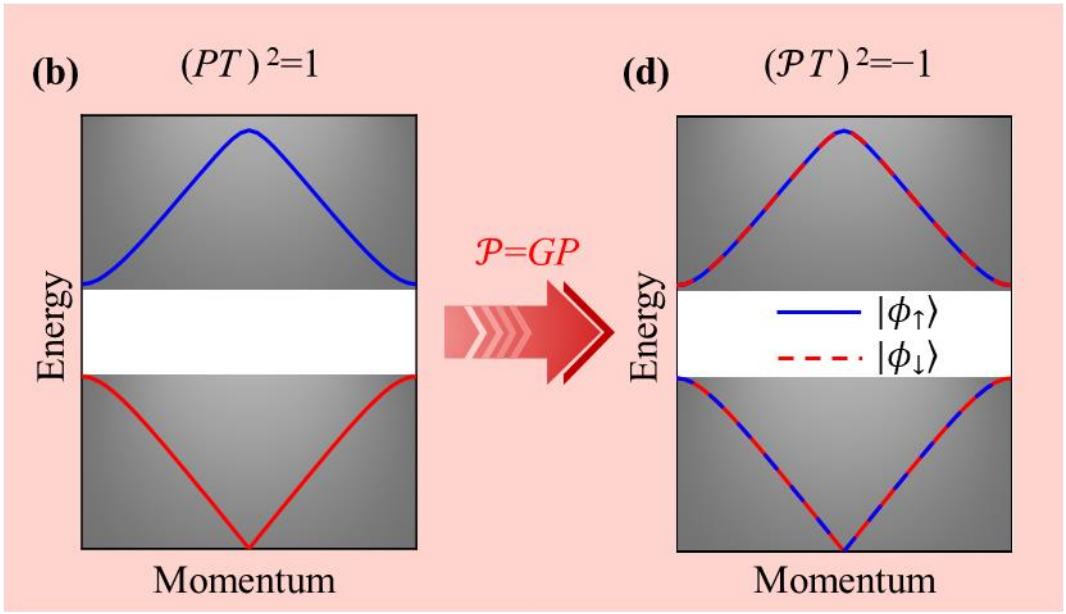
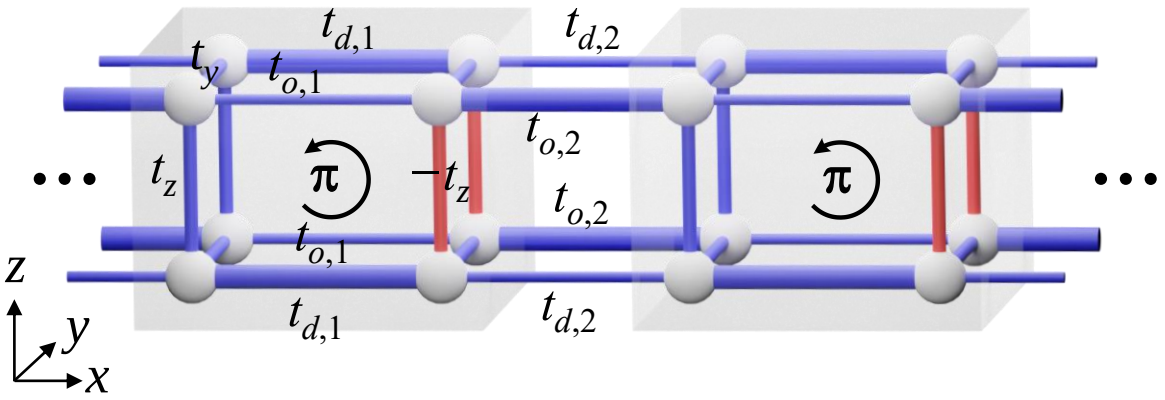
Phys. Rev. Lett.
126, 196402 (2021)





Analytical model for spinful topological phases

Tight-binding model



Hamiltonian:

$$\mathcal{H} = t_y\Gamma_{100} + t_z\Gamma_{301} + \sum_{\{s=d,o\}} \begin{bmatrix} 0 & u_s \\ u_s^* & 0 \end{bmatrix} \otimes M_s$$

$\Gamma_{\mu\nu\lambda} = \rho_\mu \otimes \tau_\nu \otimes \sigma_\lambda$,
$M_d = \mathrm{diag}(1,0,0,1)$, $M_o = \mathrm{diag}(0,1,1,0)$,
$u_d = t_{d,1} + t_{d,2}e^{-ik}$, and $u_o = t_{o,1} + t_{o,2}e^{-ik}$

Symmetry operators and $\mathbb{Z}_2$ G:

$\hat{P} = \Gamma_{111}\hat{I}$, $\hat{T} = \hat{\mathcal{K}}$, $\hat{S} = \Gamma_{333}\hat{I}$
$G = \Gamma_{003}$, $\mathcal{P} = GP$.

Topological classification:

$(\mathcal{P}T)^2\mathcal{H} = (GPT)^2\mathcal{H} = -\mathcal{H}$ $\quad (\mathcal{P}T)^2 = -1$

$(\mathcal{P}TS)\mathcal{H} + (S\mathcal{P}T)\mathcal{H} = 0,$ $\quad \{S, \mathcal{P}T\} = 0$

$$U\mathcal{H}U^\dagger = \begin{bmatrix} 0 & Q(k)_{4\times4} \\ Q^\dagger(k)_{4\times4} & 0 \end{bmatrix},$$

$$\nu = \frac{1}{2\pi i}\oint dk\ \mathrm{tr}Q^{-1}(k)\partial_k Q(k)\,. \quad \nu \in 2\mathbb{Z}$$

⇒ DIII

Spin

Eigenstates:
ψ_1 and ψ_2

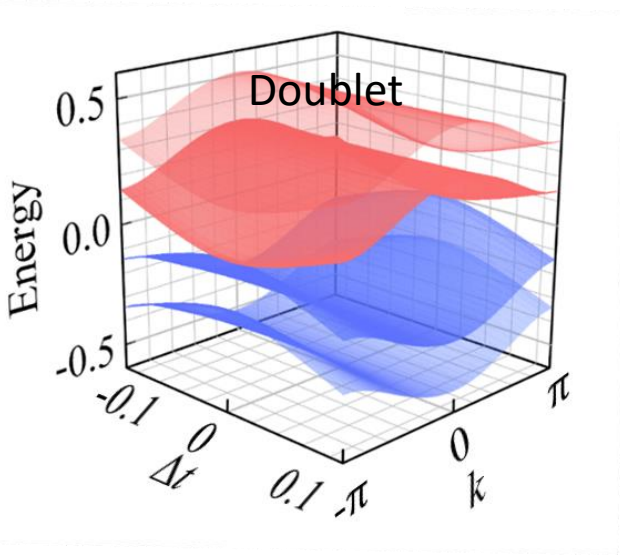
Spins:
$\psi_+ = U\psi_1$ and $\psi_- = U\psi_2$, $U = e^{(i\pi/4)\Gamma_{100}}e^{-(i\pi/4)\Gamma_{133}}$

Pseudo time-reversal symmetry $\mathcal{P}T$:

$$\psi_+ \xrightarrow{\mathcal{P}T} \psi_- \xrightarrow{\mathcal{P}T} -\psi_+ \xrightarrow{\mathcal{P}T} -\psi_-$$

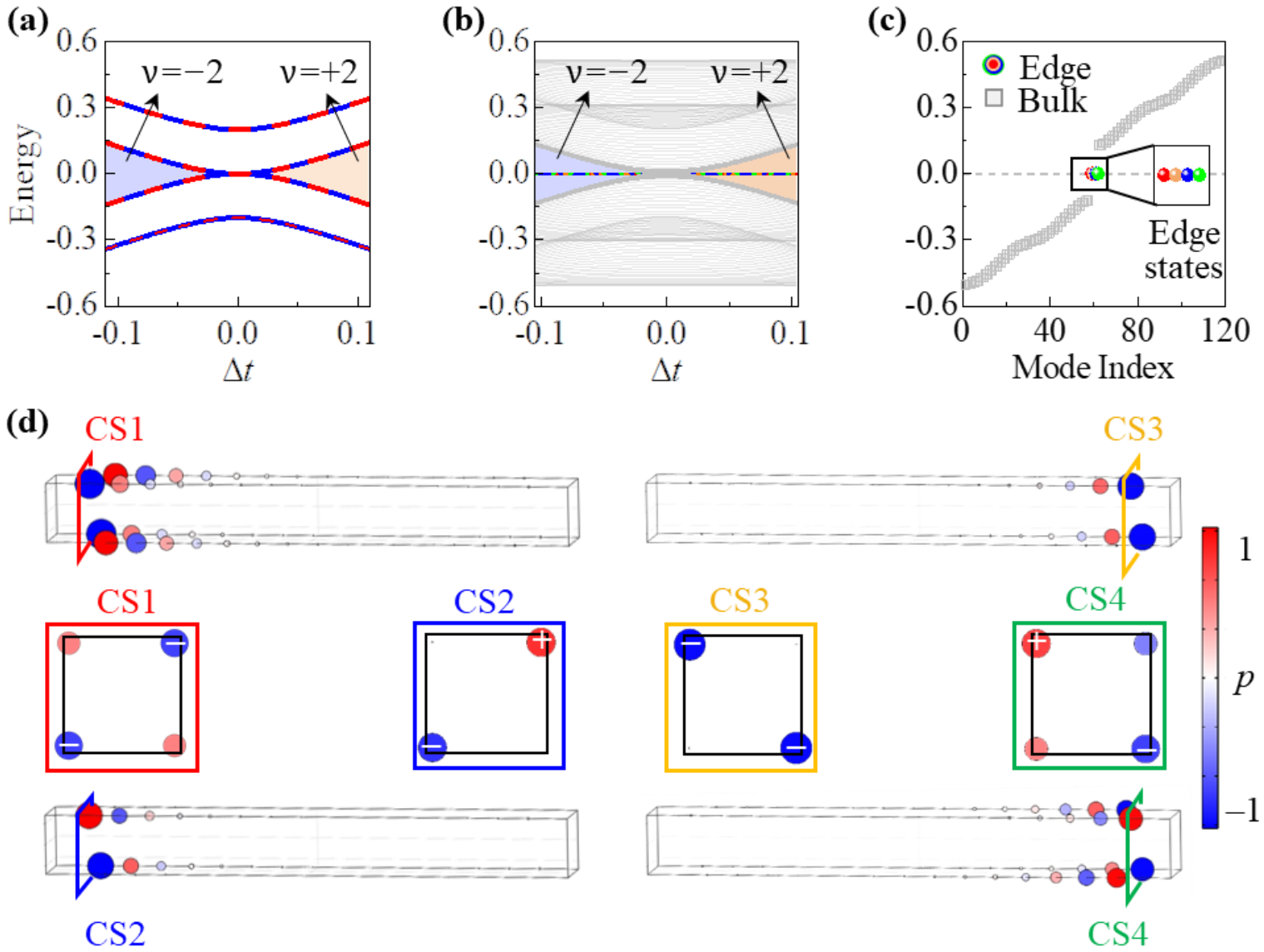
Analytical model for spinful topological phases

Spinful topological phase transition:



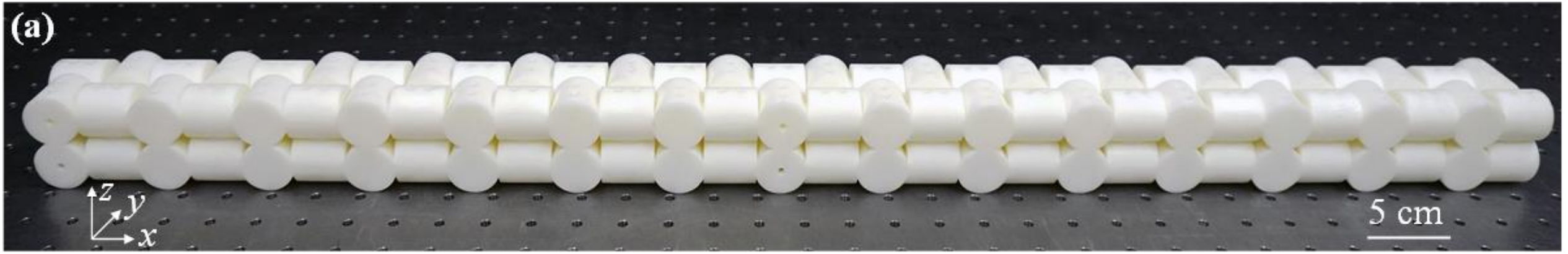
Coupling contrast:

$$\Delta t = \frac{t_{d,1} - t_{d,2}}{2}$$

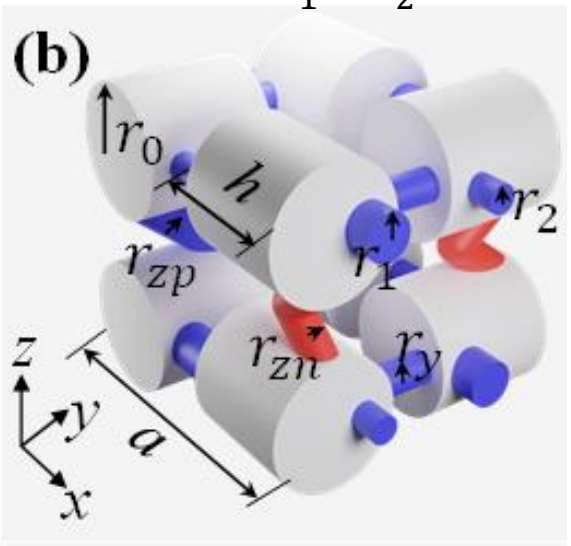


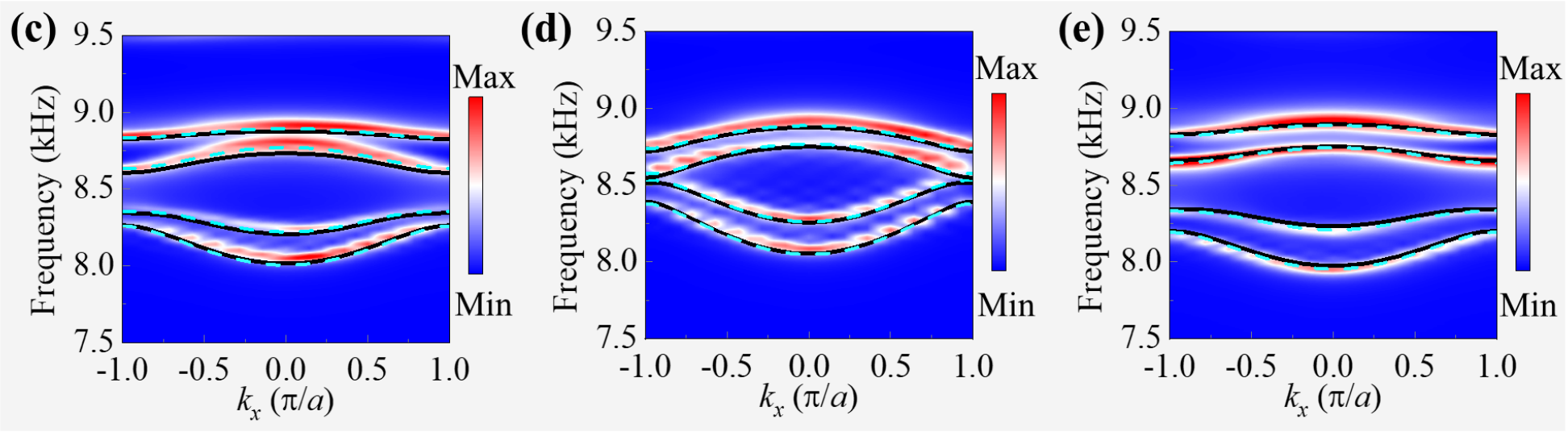
Sample design and experimental demonstration

Experimental measurement of Kramers doublet



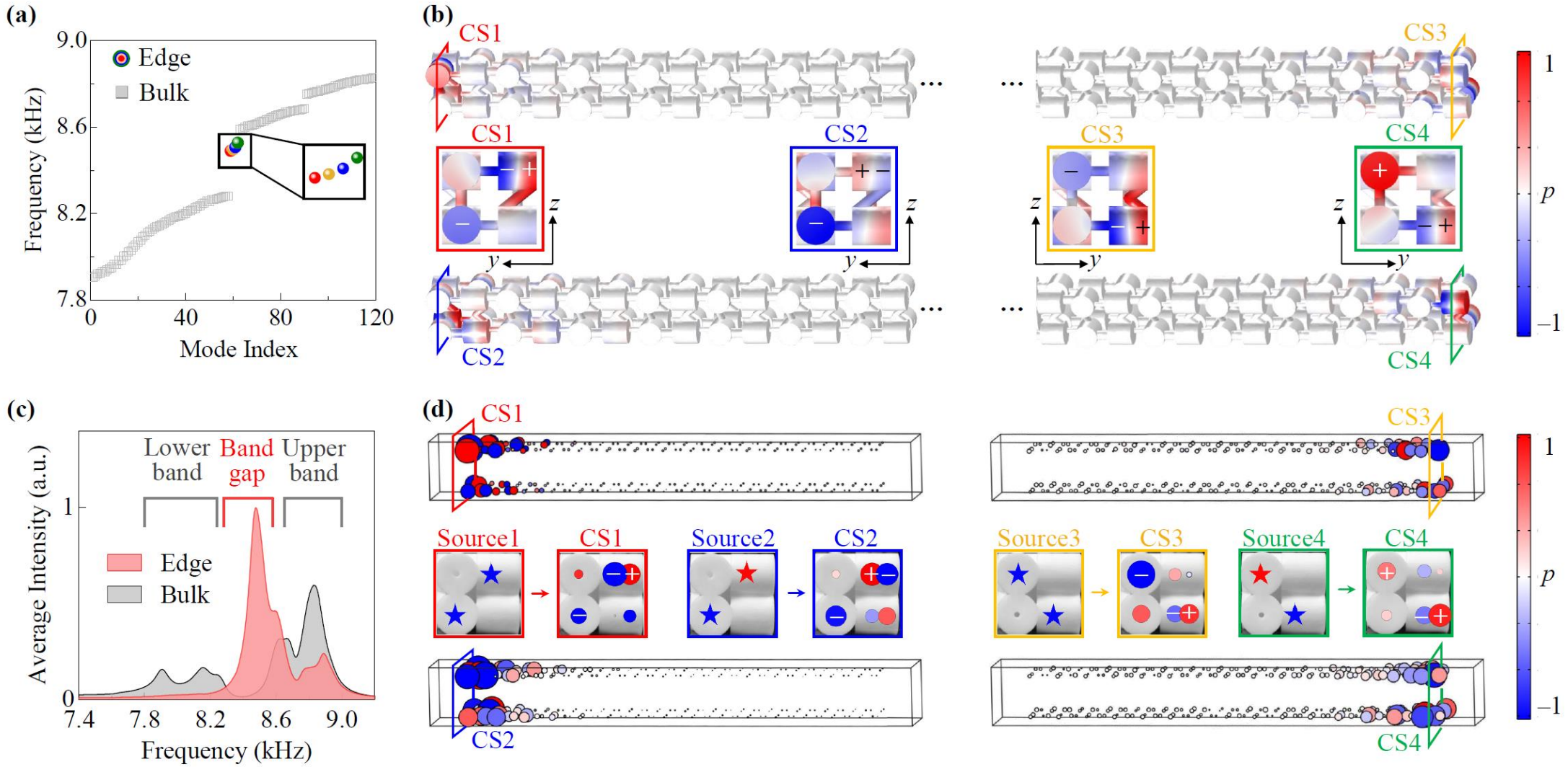
$\Delta r = r_1 - r_2$ $\Delta r = -2.2$ mm $\Delta r = 0$ $\Delta r = 2.2$ mm





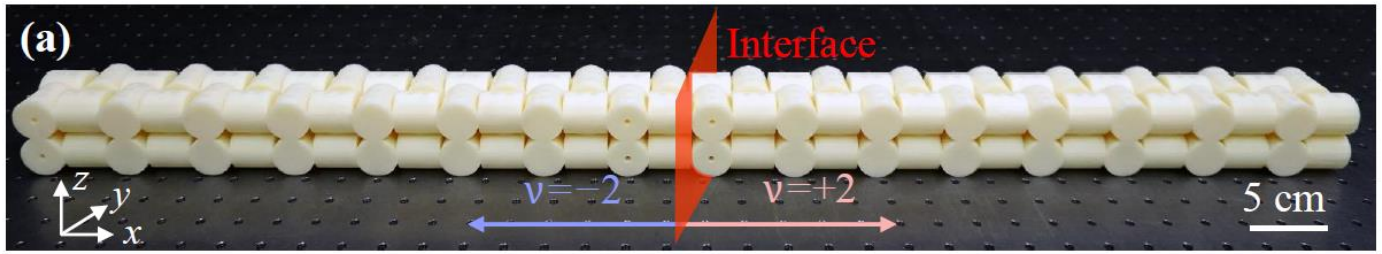
Sample design and experimental demonstration

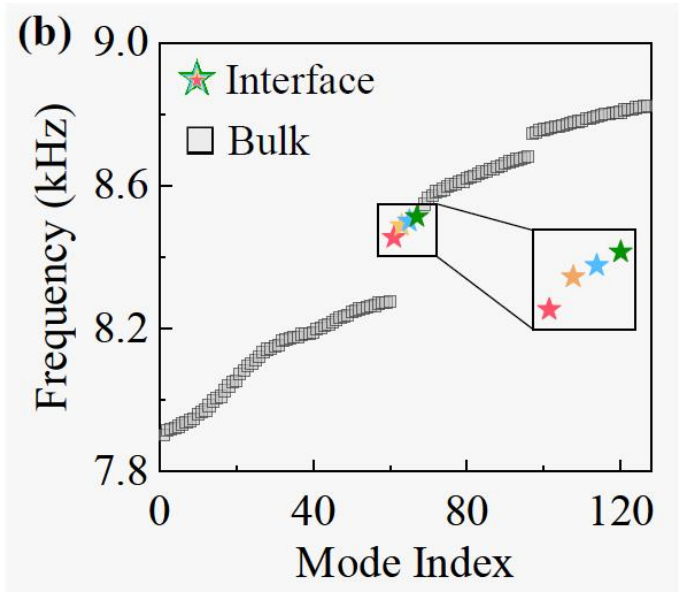
Observation of Kramers pairs of topological boundary modes

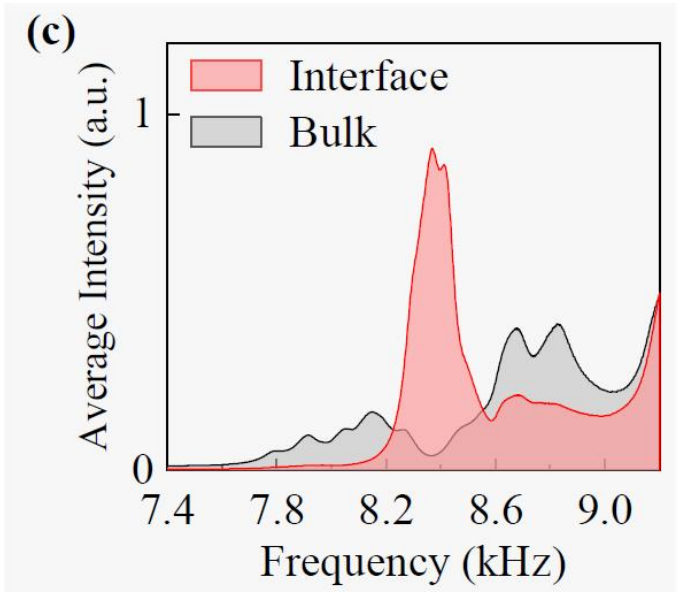


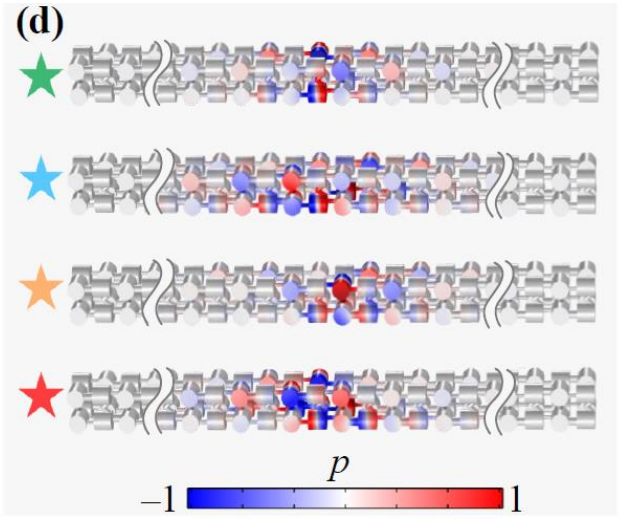
Sample design and experimental demonstration

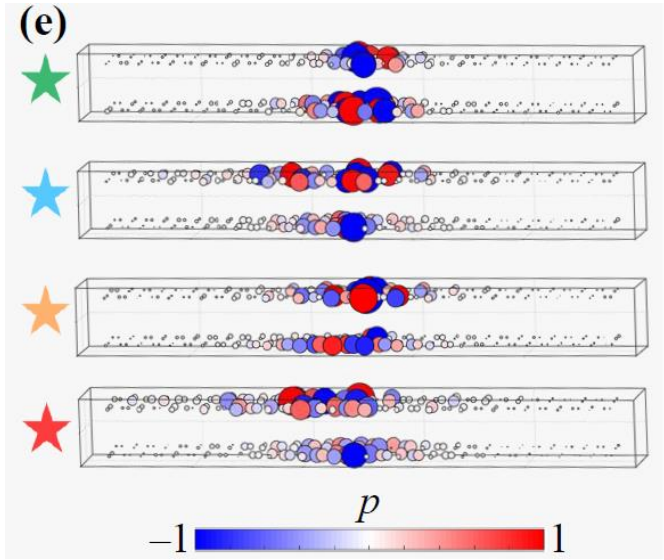
Topological interface between $\nu = +2$ and $\nu = -2$











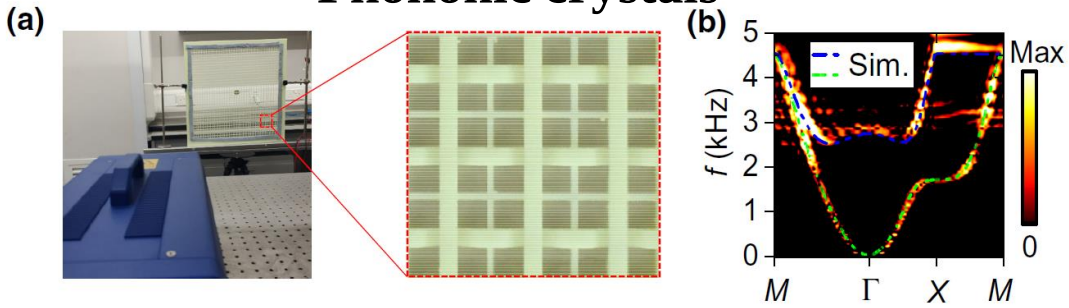
Conclusions

- **Designed a 1D topological acoustic crystals with a $\mathbb{Z}_2$ gauge field**
- **Observed the Kramers double-degenerate band structures**
- **Observed the Kramers pairs of topological boundary modes**
- **Observed the topological interface states between two spinful topological acoustic crystals with opposite winding numbers**

Y. Meng et al, PRL **130**, 026101 (2023)

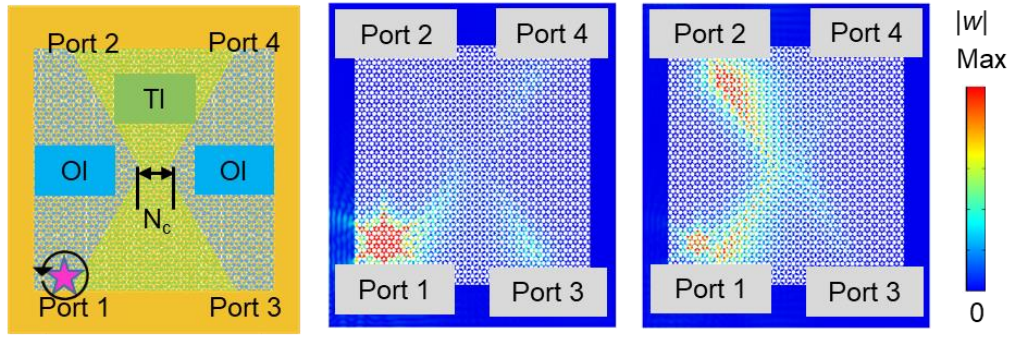
Other related works

Phononic crystals



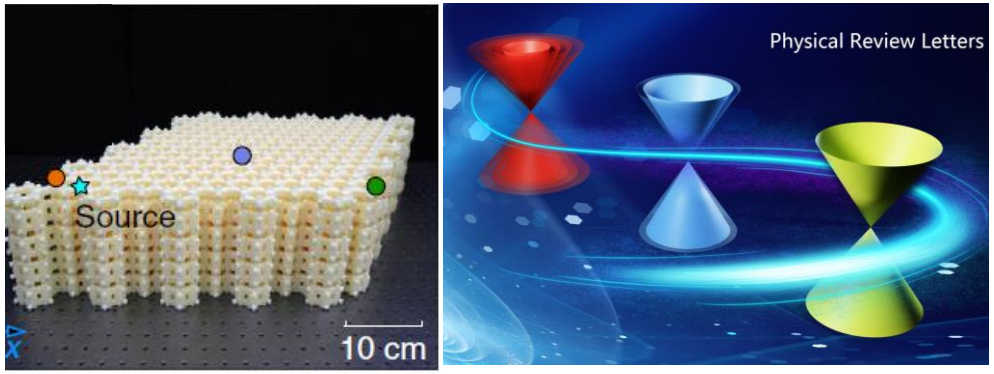
Phys. Rev. Appl. **18,** 064073 (2022)

Phononic crystals



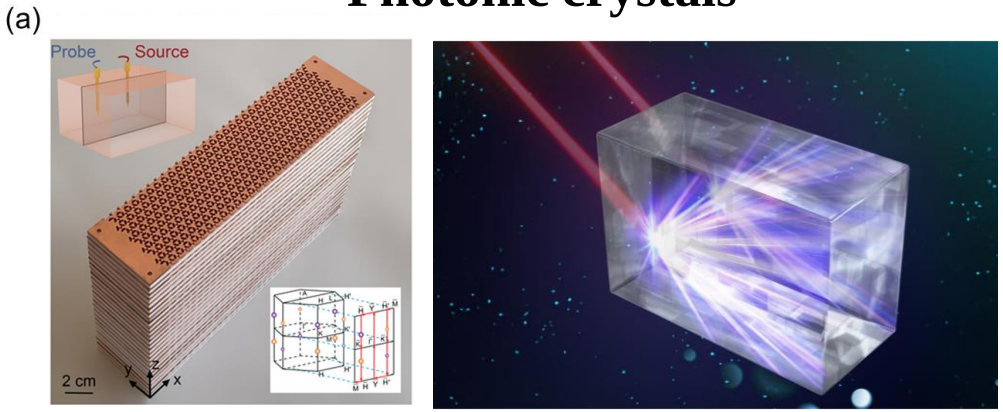
Sci. China Phys. Mech. Astron. **65**, 224611 (2022)

Acoustic crystals



Linyun Yang, Phys. Rev. Lett. **129**, 125502 (2022). (Editor's suggestion)

Photonic crystals



Xiang Xi, et al, Nat. Commun. **14** 1991 (2023)

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THANKS

Presentor: Yan MENG